Stochastic partial differential equations and measure-valued branching processes

Lecturer: Leonid Mytnik
Telephone: 4432 E-mail: leonid@ie.technion.ac.il
Time and place: Thursday, 15:30, Bloomfield 310.

Measure-valued branching processes (superprocesses) arise as limits of branching particle systems undergoing random migration and critical (or asymptotically critical) branching. In low dimensions these processes could be described as solutions to stochastic partial differential equations (SPDEs). We will deal with both superprocesses and SPDEs in this course. We will need a certain amount of machinery: characterization of processes via martingale problems, weak convergence of stochastic processes with values in metric space, martingale measures. We develop this as we need it.

As time permits we will cover some or all of the following topics.

1. Introduction: SPDEs, superprocesses and approximating particle pictures, log-Laplace equation.
2. Branching processes. Kolmogorov, Yaglom theorems. Feller diffusion.
3. Martingale problem. Examples. Existence and uniqueness.
4. Weak convergence of processes.
5. Existence of measure-valued branching processes (superprocess) via weak convergence. Uniqueness of solution to the corresponding martingale problem.
6. Regularity properties of super-Brownian motion. Fixed time properties.
7. Infinitely divisible random measures. Cluster representation of superprocesses.
8. Martingale measures. SPDEs. Characterization of super-Brownian motion as a solution to an SPDE in dimension $d = 1$.
9. Weak uniqueness for some SPDEs via duality argument.
10. SPDEs driven by Levy noise.
11. Catalytic super-Brownian motion.
12. Interacting particle system converging to superprocesses and SPDEs (contact, voter models).

References We shall not be following any particular book. The following are useful references for different aspects of the course.

1. D. Dawson. Infinitely Divisible Random Measures and Superprocesses. In H. Körezlioğlu and A. Üstünel, editors, *Stochastic Analysis and Related Topics*, Birkhäuser Boston, 1992.

2. D. Dawson. Measure-valued Markov Processes. *École d'été de Probabilités de Saint Flour*, 1991.

3. A.M. Etheridge. *An Introduction to Superprocesses*, volume 20 of *Univ. Lecture Series*. AMS, Rhode Island, 2000.

4. J. F. Le Gall. *Spatial Branching Processes, Random Snakes and Partial Differential Equations*. Birkhäuser, ETH Zürich, 1999.

5. E. Perkins. Dawson-Watanabe Superprocesses and Measure-valued Diffusions. *Lectures on probability theory and statistics (Saint-Flour, 1999),* Lecture Notes in Mathematics, 1781:125–329, 2002.

6. J. Walsh. An introduction to stochastic partial differential equations. *Lecture Notes in Mathematics*, 1180:265–439, 1986.