Chapter 28

Basic Financial Tools: A Review

The building blocks of finance include the time value of money, risk and its relationship with rates of return, and stock and bond valuation models. These topics are covered in introductory finance courses, but because of their fundamental importance, we review them in this chapter.[1]

Time Value of Money

Time value concepts, or **discounted cash flow analysis,** underlie virtually all the important topics in financial management, including stock and bond valuation, capital budgeting, cost of capital, and the analysis of financing vehicles such as convertibles and leasing. Therefore, an understanding of time value concepts is essential to anyone studying financial management.

Future Values

An investment of PV dollars today at an interest rate of i percent for n periods will grow over time to some **future value (FV).** The following **time line** shows how this growth occurs:

0 (i%)	1	2	3	. . .	n − 1	n
PV	FV_1	FV_2	FV_3		FV_{n-1}	FV_n
	$= PV(1+i)$	$= FV_1(1+i)$	$= FV_2(1+i)$		$= FV_{n-2}(1+i)$	$= FV_{n-1}(1+i)$

This process is called **compounding,** and it can be expressed with the following equation:

$$FV_n = PV(1+i)^n = PV(FVIF_{i,n}). \tag{28-1}$$

The term $\mathbf{FVIF_{i,n}}$ is called the **future value interest factor.**

The future value of a series of cash flows is the sum of the future values of the individual cash flows. An **ordinary annuity** has equal payments, with

[1] This review is limited to material that is necessary to understand the chapters in the main text. For a more detailed treatment of risk, return, and valuation models, see Chapters 2 through 5. For a more detailed review of time value of money concepts, see Eugene F. Brigham and Michael C. Ehrhardt, *Financial Management*, 10th edition (Cincinnati, OH: South-Western, 2001), or some other introductory finance text.

symbol PMT, that occur at the end of each period, and its future value (FVA_n) is found as follows:

0	1	2	3	. . .	n − 1	n
	PMT	PMT	PMT		PMT	PMT

$$\begin{aligned} \text{Future value} = FVA_n &= PMT(1 + i)^{n-1} + PMT(1 + i)^{n-2} + \cdots \\ &\quad + PMT(1 + i) + PMT \\ &= PMT\left[\frac{(1 + i)^n - 1}{i}\right] \\ &= PMT(FVIFA_{i,n}). \end{aligned} \tag{28-2}$$

The second and third forms of Equation 28-2 represent more convenient ways to solve the equation set forth on the first row.[2] The term **$FVIFA_{i,n}$** is called the **future value interest factor of an annuity** at i percent for n periods.[3]

Here are some applications of these concepts. First, consider a single payment, or lump sum, of \$500 made today. It will earn 7 percent per year for 25 years. This \$500 present value will grow to \$2,713.72 after 25 years:

$$\text{Future value} = \$500(1 + 0.07)^{25} = \$500(5.42743) = \$2{,}713.72. \tag{28-1a}$$

Now suppose we have an annuity with 25 annual payments of \$500 each, starting a year from now, and the interest rate is 7 percent per year. The future value of the annuity is \$31,624.52:

$$\text{Future value} = \$500\left[\frac{(1 + 0.07)^{25} - 1}{0.07}\right] = \$500(63.24904) = \$31{,}624.52. \tag{28-2a}$$

A financial calculator could be used to solve this problem. On most calculators, the N button is for the number of periods. We recommend setting the calculator to one period per year, with payments occurring at the end of the year. The I (or I/Y) button is for the interest rate as a percentage, not as a decimal. The PV button is for the value today of the future cash flows, the FV button is for a lump sum cash flow at the end of N periods, and the PMT button is used if we have a series of equal payments that occur at the end of each period. On some calculators the CPT button is used to compute present and future values, interest rates, and payments. Other calculators have different ways to enter data and find solutions, so be sure to check your specific manual.[4] To find the future value of the single payment in the example above, input N = 25, I = 7, PV = −500 (negative because it is a cash outflow), and PMT = 0 (because we have no recurring payments). Press CPT and then the FV key to find FV = \$2,713.72. To calculate the future value of our annuity, input N = 25, I = 7, PV = 0, PMT = −500, and then press CPT

[2] See the Extension to Chapter 5 for a derivation of the sum of a geometric series.

[3] An annuity in which payments are made at the start of the period is called an **annuity due.** The future value interest factor of an annuity due is $(1+i)FVIFA_{i,n}$.

[4] Our *Technology Supplement* contains tutorials for the most commonly used financial calculators (about 12 typewritten pages versus much more for the calculator manuals). Our tutorials explain how to do everything needed in this book. See our Preface for information on how to obtain the *Technology Supplement*.

and then the FV key to find FV = $31,624.52. Some financial calculators will display the negative of this number.

Present Values

The value today of a future cash flow or series of cash flows is called the **present value (PV).** The present value of a lump sum future payment, FV_n, to be received in n years and **discounted** at the interest rate i, is

$$PV = FV_n \frac{1}{(1+i)^n} = FV_n(PVIF_{i,n}). \tag{28-3}$$

$PVIF_{i,n}$ is the **present value interest factor** at i percent due in n periods.

The **present value of an annuity** is the sum of the present values of the individual payments. Here is the time line and formula for an ordinary annuity:

$$\text{Present value} = PVA_n = \frac{PMT}{(1+i)} + \frac{PMT}{(1+i)^2} + \cdot\cdot\cdot + \frac{PMT}{(1+i)^{n-1}} + \frac{PMT}{(1+i)^n}$$
$$= PMT\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$
$$= PMT(PVIFA_{i,n}). \tag{28-4}$$

$PVIFA_{i,n}$ is the **present value interest factor for an annuity** of i percent for n periods.[5]

The present value of a $500 lump sum to be received in 25 years when the interest rate is 7 percent is $\$500/(1.07)^{25} = \$92.12$. The present value of a series of 25 payments of $500 each discounted at 7 percent is $5,826.79:

$$PV = \$500\left(\frac{1 - \frac{1}{(1.07)^{25}}}{0.07}\right) = 500(11.65358)$$
$$= \$5{,}826.79. \tag{28-4a}$$

Present values can be calculated using a financial calculator. For the lump sum payment, enter N = 25, I = 7, PMT = 0, and FV = −500, and then press CPT and then the PV key to find PV = $92.12. For the annuity, enter N = 25, I = 7, PMT = −500, and FV = 0, and then press CPT and then the PV key to find PV = $5,826.79.

Present values and future values are directly related to one another. We saw just above that the present value of $500 to be received in 25 years is

[5] The present value interest factor of an annuity due is $(1 + i)PVIFA_{i,n}$.

\$92.12. This means that if we had \$92.12 now and invested it at a 7 percent interest rate, it would grow to \$500 in 25 years. For the annuity example, if you put \$5,826.79 in an account earning 7 percent, then you could withdraw \$500 at the end of each year for 25 years, and have a balance of zero at the end of 25 years.

An important application of the annuity formula is finding the set of equal payments necessary to amortize a loan. In an amortized loan (such as a mortgage or an auto loan) the payment is set so that the present value of the series of payments, when discounted at the loan rate, is equal to the amount of the loan:

$$\text{Loan amount} = \text{PMT}(\text{PVIFA}_{i,n}), \text{ or}$$
$$\text{PMT} = (\text{Loan amount})/\text{PVIFA}_{i,n}.$$

Each payment consists of two elements: (1) interest on the outstanding balance (which changes over time) and (2) a repayment of principal (which reduces the loan balance). For example, consider a 30-year, \$200,000 home mortgage with monthly payments and a nominal rate of 9 percent per year. There are 30(12) = 360 monthly payments, and the monthly interest rate is 9%/12 = 0.75%. We could use Equation 28-4 to calculate $\text{PVIFA}_{0.75\%,360}$, and then find the monthly payment, which would be

$$\text{PMT} = \$200{,}000/\text{PVIFA}_{0.75\%,360} = \$1{,}609.25.$$

It would be easier to use a financial calculator, entering N = 360, I = 0.75, PV = −200000, and FV = 0, and then press CPT and then the PMT key to find PMT = \$1,609.25.

To see how the loan is paid off, note that the interest due in the first month is 0.75 percent of the initial outstanding balance, or 0.0075 (\$200,000) = \$1,500.00. Since the total payment was \$1,609.25, then \$1,609.25 − 1,500.00 = \$109.25 is applied to reduce the principal balance. At the start of the second month the outstanding balance would be \$200,000 − \$109.25 = \$199,890.75. The interest on this balance would be 0.75 percent of the new balance or 0.0075(\$199,890.75) = \$1,499.18, and the amount applied to reduce the principal would be \$1,609.25 − 1,499.18 = \$110.07. This process would be repeated each month, and the resulting **amortization schedule** would show, for each month, the amount of the payment that is interest and the amount applied to reduce principal. With a spreadsheet program such as *Excel,* we can easily calculate amortization schedules.

Nonannual Compounding

Not all cash flows occur once a year. The periods could be years, quarters, months, days, hours, minutes, seconds, or even instantaneous periods.

DISCRETE COMPOUNDING The procedure used when the period is less than a year is to take the annual interest rate, called the **nominal,** or **quoted rate,** and divide it by the number of periods in a year. The result is called the **periodic rate.** In the case of a monthly annuity with a nominal annual rate of 7 percent, the monthly interest rate would be 7%/12 = 0.5833%. As a decimal, this is 0.005833. Note that interest rates are sometimes stated as decimals and sometimes as percentages. You must be careful to determine which form is being used.

When interest is compounded more frequently than once a year, interest will be earned on interest more frequently. Consequently, the effective rate will exceed the quoted rate. For example, a dollar invested at a quoted (nominal) annual rate of 7 percent but compounded monthly will earn 7/12 = 0.5833 percent per month for 12 months. Thus, \$1 will grow to $\$1(1.005833)^{12} = 1.0723$ over one year, or by 7.23%. Therefore, the **effective,** or **equivalent, annual rate (EAR)** on a 7 percent nominal rate compounded monthly is 7.23 percent.

If there are m compounding intervals per year and the nominal rate is i_{nom}, then the effective annual rate will be

$$\text{EAR (or EFF\%)} = \left(1 + \frac{i_{nom}}{m}\right)^m - 1.0. \tag{28-5}$$

The larger the value of m, the greater the difference between the nominal and effective rates. Note that if m = 1, which means annual compounding, then the nominal and effective rates are equal.[6]

Solving for the Interest Rate

The general formula for the present value of a series of cash flows, CF_t, discounted at some rate i, is as follows:

$$PV = \frac{CF_1}{1 + i} + \frac{CF_2}{(1 + i)^2} + \cdot \cdot \cdot + \frac{CF_n}{(1 + i)^n} = \sum_{t=1}^{n} \frac{CF_t}{(1 + i)^t}. \tag{28-6}$$

The cash flows can be equal, in which case $CF_t = PMT =$ constant, so we have an annuity, or the CF_t can vary from period to period. Now suppose we know the current price of the asset, which is by definition the PV of the cash flows, and the expected cash flows themselves, and we want to find the rate of return on the asset, or its yield. The asset's expected rate of return, or yield, is defined to be the value of i that solves Equation 28-6. For example, suppose you plan to finance a car that costs \$22,000, and the dealer offers a 5-year payment plan that requires \$2,000 down and payments of

[6] For some purposes it is appropriate to use instantaneous, or continuous, compounding. Equation 28-5 can be used for any number of compounding intervals, m, per year: m = 1 (annual compounding), m = 2 (semiannual compounding), m = 12 (monthly compounding), m = 365 (daily compounding), m = 365(24) = 8,760 (hourly compounding), and even more frequently. As m approaches infinity, the compounding interval approaches zero, and the compounding becomes **instantaneous,** or **continuous.** The EAR for continuous compounding is given by the following equation:

$$EAR_{continuous} = e^{i_{nom}} - 1.0. \tag{28-5a}$$

Here e is approximately equal to 2.71828; most calculators have a special key for e. The EAR of a 7 percent investment compounded continuously is $e^{0.07} - 1.0 = 0.0725 = 7.25\%$.

Equation 28-5a can also be used to find the future value of a payment invested for n years at a rate i_{nom} compounded continuously:

$$FV_n = PV(e^{i_{nom}n}). \tag{28-1a}$$

Similarly, the present value of a future sum to be received in n periods discounted at a nominal rate of i_{nom} compounded continuously is found using this equation:

$$PV = \frac{FV_n}{e^{i_{nom}n}} = FV_n(e^{-i_{nom}n}). \tag{28-2a}$$

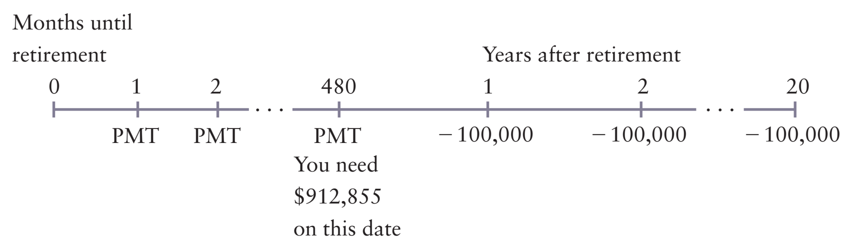
\$415.17 per month for 60 months. Since you would be financing \$20,000 over 60 months, the monthly interest rate on the loan is the value of i that solves Equation 28-4a:

$$\$20{,}000 = 415.17(\text{PVIFA}_{i,60}). \tag{28-4a}$$

This equation would be difficult to solve by hand, but it is easy with a financial calculator. Enter PV = 20000, PMT = −415.17, FV = 0, and N = 60, and press CPT and then the I key to find I = 0.75, or ¾ percent a month. The nominal annual rate is 12(0.0075) = 0.09 = 9%, and the effective annual rate is $(1.0075)^{12} - 1 = 9.38\%$.

Complex Time Value Problems

The present and future value equations can be combined to find the answers to more complicated problems. For example, suppose you want to know how much you must save each month to retire in 40 years. After retiring, you plan to withdraw \$100,000 per year for 20 years, with the first withdrawal coming one year after retirement. You will put away money at the end of each month, and you expect to earn 9 percent on your investments. Here is a diagram of the problem:



The solution requires two steps. First, you must find the amount needed to fund the 20 retirement payments of \$100,000. This amount is found as follows: $\text{PV} = \$100{,}000(\text{PVIFA}_{9\%,20}) = \$100{,}000(9.12855) = \$912{,}855$. Therefore, you must accumulate \$912,855 by the $40(12) = 480^{\text{th}}$ month if you are to make the 20 withdrawals after retirement. Each of the 480 deposits will earn 9%/12 = 0.75% per month. The second step is to find your required monthly payment. This involves finding the PMT stream that grows to the required future value, and it is found as follows:

$$\text{FV} = \$912{,}855 = \text{PMT}(\text{FVIFA}_{0.75\%,\,480}) = \text{PMT}(4{,}681.32).$$
$$\text{PMT} = \$912{,}855/4{,}681.32 = \$195.00 \text{ per month.}$$[7]

The problem could also be solved with a financial calculator. First, to calculate the present value of the 20 payments of \$100,000 at an annual interest rate of 9 percent, enter N = 20, I = 9, PMT = −100000, and FV = 0, and then press CPT and then the PV key to find PV = \$912,855. Second, clear

[7] For simplicity we have assumed that your retirement funds will earn 9 percent compounded *monthly* during the accumulation phase and 9 percent compounded *annually* during the withdrawal phase. It is probably more realistic to assume that your earnings will be compounded the same during both phases. In that case you would use the effective annual rate for 9 percent compounded monthly, which is 9.38 percent, as the annual interest rate during the withdrawal phase. This larger effective rate would reduce the amount you need to save by the time you retire to \$888,620, and your monthly contribution would be \$189.82.

Note also that it is relatively easy to solve problems such as this with a spreadsheet program such as *Excel*. Moreover, with a spreadsheet model, you could make systematic changes in variables such as the interest rate earned, years to retirement, years after retirement, and the like, and determine how sensitive the monthly payments are to changes in these variables. This is especially useful to financial planners.

the calculator and then enter FV = 912,855, N = 480, PV = 0, and I = 9/12, and then press CPT and then the PMT key to find PMT = −195.00, which is the monthly investment required to accumulate a balance of $912,855 in 40 years. Note that the $912,855 is both a present value and a future value in this problem. It is a present value when finding the accumulated amount required to provide the post-retirement withdrawals, but it is a future value when used to find the pre-retirement payments.

SELF-TEST QUESTIONS

What is compounding? How is compounding related to discounting?

Explain how financial calculators can be used to solve present value and future value problems.

What is the difference between an ordinary annuity and an annuity due?

Why is semiannual compounding better than annual compounding from a saver's standpoint? What about from a borrower's standpoint?

Define the terms "effective (or equivalent) annual rate," "nominal interest rate," and "periodic interest rate."

How would one construct an amortization schedule?

BOND VALUATION

Finding the value of a bond is a straightforward application of the discounted cash flow process. A bond's **par value** is its stated face value, which is the amount the issuer must pay to the bondholder at maturity. We assume a par value of $1,000 in all our examples, but it is possible to have any value that is a multiple of $1,000. The **coupon payment** is the periodic interest payment that the bond provides. Usually these payments are made every six months (semiannual payments), and the total annual payment as a percentage of the par value is called the **coupon interest rate.** For example, a 15-year, 8 percent coupon, $1,000 par value bond with semiannual payments calls for a $40 payment each six months, or $80 per year, for 15 years, plus a final principal repayment of $1,000 at maturity. A time line can be used to diagram the payment stream:

0.5 year	0	1	2	3		28	29	30
Year	0	0.5	1.0	1.5	. . .	14	14.5	15
		$40	$40	$40		$40	$40	$1,040

The last payment of $1,040 consists of a $40 coupon plus the $1,000 par value.

Bond Prices

The price of a bond is the present value of its payments, discounted at the current market interest rate appropriate for the bond, given its risk, maturity, and other characteristics. The semiannual coupons constitute an annuity, and the final principal payment is a lump sum, so the price of the bond can be found as the present value of an annuity plus the present value of a lump sum. If the market interest rate for the bond is 10 percent, or 5 percent per six months, then the value of the bond is[8]

[8] By convention, annual coupon rates and market interest rates on bonds are quoted at two times their six-month rates. So, a $1,000 bond with a $45 semiannual coupon payment has a nominal annual coupon rate of 9 percent.

$$P = \sum_{t=1}^{N} \frac{\text{Coupon}}{(1 + r_d)^t} + \frac{\text{Par}}{(1 + r_d)^N} \tag{28-7}$$

$$= \sum_{t=1}^{30} \frac{40}{1.05^t} + \frac{1000}{1.05^{30}} = \$846.28.$$

This value can be calculated with a financial calculator. There are 30 six-month periods, so enter N = 2(15) = 30, I = 10/2 = 5, PMT = 40, and FV = 1000, and then press CPT and then the PV key to find PV = −846.28. Therefore, you should be willing to pay $846.28 to buy the bond. This bond is selling at a **discount—**its price is less than its par value. This probably occurs because interest rates increased since the bond was issued. If the market interest rate fell to 6 percent, then the price of the bond would increase to

$$P = \sum_{t=1}^{30} \frac{40}{1.03^t} + \frac{1000}{1.03^{30}} = \$1{,}196.00.$$

Thus, the bond would trade at a **premium,** or above par. If the interest rate were exactly equal to the coupon rate, then the bond would trade at $1,000, or **at par.** In general,

1. If the going interest rate is *greater* than the coupon rate, the price of the bond will be less than par.
2. If the going interest rate is *less* than the coupon rate, the price will be greater than par.
3. If the going interest rate is *equal* to the coupon rate, the bond will sell at its par value.

Yield to Maturity

We used Equation 28-7 to find the price of a bond given market interest rates. We can also use it to find the interest rate given the bond's price. This interest rate is called the **yield to maturity,** and it is defined as the discount rate that sets the present value of all of the cash flows until maturity equal to the bond's current price. Financial calculators are used to make this calculation. For example, suppose our 15-year, 8 percent coupon, semiannual payment, $1,000 par value bond is selling for $925. Because the price is less than par, the yield on the bond must be greater than 8 percent, but by how much? Enter N = 30, PV = − 925, PMT = 40, and FV = 1000, and then press CPT and then the I key to find I = 4.458. This is the rate per 6 months, so the annual yield is 2(4.458) = 8.916%.[9]

Price Sensitivity to Changes in Interest Rates

Equation 28-7 also shows that a bond's price depends on the market interest rate used to discount cash flows. Therefore, fluctuations in interest rates give rise to changes in bond prices, and this sensitivity is called **interest rate risk.** To illustrate, consider an 8 percent, $1,000 par value U.S. Treasury bond with a 30-year maturity and semiannual payments when the going market

[9] By convention, a bond with a yield of 4.458% per six months is said to have a nominal annual yield of 2(4.458%) = 8.916%. Also, for most calculators PV must be opposite in sign to both PMT and FV in order to calculate the yield. However this convention may differ across calculators, so check your user manual for your particular calculator to make sure.

interest rate is 8 percent. Since the coupon rate is equal to the interest rate, the bond sells at par, or for $1,000. If market interest rates increase, then the price of the bond will fall. If the rate increases to 10 percent, then the new price will be $810.71, so the bond will have fallen by 18.9 percent. Notice that this decline has nothing to do with the riskiness of the bond's coupon payments—even the prices of risk-free U.S. Treasury bonds fall if interest rates increase.

The percentage price change in response to a change in interest rates depends on (1) the maturity of the bond and (2) its coupon rate. Other things held constant, a longer-term bond will experience larger price changes than a shorter-term bond, and a lower coupon bond will have a larger change than a higher coupon bond. To illustrate, if our 8 percent Treasury bond had a 5-year rather than a 30-year maturity, then the new price after an interest rate increase from 8 percent to 10 percent would be $922.78, a 7.7 percent decrease versus the 18.9 percent decrease for the 30-year bond. Because the shorter-term bond has less price risk, this bond is said to have less interest rate risk than the longer-term bond.

A bond's interest rate risk also depends on its coupon—the lower the coupon, other things held constant, the greater the interest rate risk. To illustrate, consider the example of a **zero coupon bond,** or simply a "**zero.**" It pays no coupons, but it sells at a discount and provides its entire return at maturity. A $1,000 par value, 30-year zero in an 8 percent annual rate market will sell for $99.38. If the interest rate increases by 2 percentage points, to 10 percent, then the bond's price will drop to $57.31, or by 42.3 percent! The zero drops so sharply because distant cash flows are impacted more heavily by higher interest rates than near-term cash flows, and all of the zero's cash flows occur at the end of its 30-year life.

These price changes are summarized below:

Bond	Initial Price @ 8%	Price @ 10%	Percent Change	Interest Rate Risk
30-year, 8% coupon	$1,000.00	$810.71	−18.9%	Significant
5-year, 8% coupon	1,000.00	922.78	−7.7	Rather low
30-year zero	99.38	57.31	−42.3	Very high

SELF-TEST QUESTIONS

Explain verbally the following equation:

$$P = \sum_{t=1}^{N} \frac{\text{Coupon}}{(1 + r_d)^t} + \frac{\text{Par}}{(1 + r_d)^N}.$$

Explain what happens to the price of a fixed-rate bond if (1) interest rates rise above the bond's coupon rate or (2) interest rates fall below the coupon rate.

What is a "discount bond"? A "premium bond"? A "par bond"?

What is "interest rate risk?" What two characteristics of a bond affect its interest rate risk?

If interest rate risk is defined as the percentage change in the price of a bond given a 10% change in the going rate of interest (e.g., from 8 percent to 8.8 percent), which of the following bonds would have the most interest rate risk? Assume that the initial market interest rate for each bond is 8 percent, and assume that the yield curve is horizontal.

(1) A 30-year, 8 percent coupon, annual payment T-bond.
(2) A 10-year, 6 percent coupon, annual payment T-bond.
(3) A 10-year, zero coupon, T-bond.

RISK AND RETURN

Risk is the possibility that an outcome will be different from what is expected. For an investment, risk is the possibility that the actual return (dollars or percent) will be less than the expected return. We will consider two types of risk for assets: **stand-alone risk** and **portfolio risk.** Stand-alone risk is the risk an investor would bear if he or she held only a single asset. Portfolio risk is the risk that an asset contributes to a well-diversified portfolio.

Statistical Measures

To quantify risk, we must enumerate the various events that can happen and the probabilities of those events. We will use **discrete probabilities** in our calculations, which means we assume a finite number of possible events and probabilities. The list of possible events, and their probabilities, is called a **probability distribution.** Each probability must be between 0 and 1, and the sum must equal 1.0. For example, suppose the long-run demand for Mercer Products' output could be strong, normal, or weak, and the probabilities of these events are 30 percent, 40 percent, and 30 percent, respectively. Suppose further that the rate of return on Mercer's stock depends on demand as shown in Table 28-1, which also provides data on another company, U.S. Water. We explain the table in the following sections.

EXPECTED RETURN The **expected rate of return** on a stock with possible returns r_i and probabilities P_i is found for Mercer with this equation:

$$\text{Expected rate of return} = \hat{r} = P_1r_1 + P_2r_2 + \cdots + P_nr_n = \sum_{i=1}^{n} P_ir_i \qquad (28\text{-}8)$$

$$= 0.30(100\%) + 0.40(15\%) + 0.30(-70\%) = 15\%.$$

Note that only if demand is normal will the actual 15 percent return equal the expected return. If demand is strong, the actual return will exceed the expected return by $100\% - 15\% = 85\% = 0.85$, so the deviation from the mean is $+0.85$ or $+85$ percent. If demand is weak, the actual return will be less than the expected return by $15\% - (-70\%) = 85\% = 0.85$, so the deviation from the mean is -0.85 or -85 percent. Intuitively, larger deviations

TABLE 28-1 Probability Distribution for Mercer Products and U.S. Water

Demand for Products	Probability	Mercer Products' Stock Return	U.S. Water's Stock Return
Strong	0.30	100.0%	20.0%
Normal	0.40	15.0	15.0
Weak	0.30	(70.0)	10.0
	1.00		
Expected return		15.0%	15.0%
Standard deviation		65.8%	3.9%

signify higher risk. Notice that the deviations for U.S. Water are considerably smaller, indicating a much less risky stock.

VARIANCE **Variance** measures the extent to which the actual return is likely to deviate from the expected value, and it is defined as the weighted average of the squared deviations:

$$\text{Variance} = \sigma^2 = \sum_{i=1}^{n} (r_i - \hat{r})^2 P_i. \tag{28-9}$$

In our example, Mercer Products' variance, using decimals rather than percentages, is $0.30(0.85)^2 + 0.40(0.0)^2 + 0.30(-0.85)^2 = 0.4335$. This means that the weighted average of the squared differences between the actual and expected returns is 0.4335 = 43.35 percent.

The variance is not easy to interpret. However, the **standard deviation,** or σ, which is the square root of the variance and which measures how far the actual future return is likely to deviate from the expected return, can be interpreted easily. Therefore, σ is often used as a measure of risk. In general, if returns are normally distributed, then we can expect the actual return to be within one standard deviation of the mean about 68 percent of the time.

For Mercer Products, the standard deviation is $\sigma = \sqrt{0.4335} = 0.658 = 65.8\%$. Assuming that Mercer's returns are normally distributed, there is about a 68 percent probability that the actual future return will be between 15% − 65.8% = −50.8% and 15% + 65.8% = 80.8%. Of course, this also means that there is a 32 percent probability that the actual return will be either less than −50.8 percent or greater than 80.8 percent.

The higher the standard deviation of a stock's return, the more standalone risk it has. U.S. Water's returns, which were also shown in Table 28-1, also have an expected value of 15 percent. However, U.S. Water's variance is only 0.0015, and its standard deviation is only 0.0387 or 3.87 percent. Therefore, assuming U.S. Water's returns are normally distributed, then there is a 68 percent probability that its actual return will be in the range of 11.13 percent to 18.87 percent. The returns data in Table 28-2 clearly indicate that Mercer Products is much riskier than U.S. Water.

COEFFICIENT OF VARIATION The **coefficient of variation**, calculated using Equation 28-10, facilitates comparisons between returns that have different expected values:

$$\text{Coefficient of variation} = \text{CV} = \frac{\sigma}{\hat{r}}. \tag{28-10}$$

Dividing the standard deviation by the expected return gives the standard deviation as a percentage of the expected return. Therefore, the CV measures

TABLE 28-2 Return Ranges for Mercer Products and U.S. Water if Returns Are Normal

Return Range	Mercer Returns	U.S. Water Returns	Probability of Return in Range
$\hat{r} \pm 1\sigma$	−50.8% to 80.8%	11.1% to 18.9%	68%

the amount of risk per unit of expected return. For Mercer, the standard deviation is over four times its expected return, and its CV is 0.658/0.15 = 4.39. U.S. Water's standard deviation is much smaller than its expected return, and its CV is only 0.0387/0.15 = 0.26. By the coefficient of variation criterion, Mercer is 17 times riskier than U.S. Water.

Portfolio Risk and Return

Most investors do not keep all of their money invested in just one asset; instead, they hold collections of assets called **portfolios.** The fraction of the total portfolio invested in an individual asset is called the asset's **portfolio weight,** w_i. The **expected return on a portfolio,** $\hat{r}_p$, is the weighted average of the expected returns on the individual assets:

$$\text{Expected return on a portfolio} = \hat{r}_p = w_1\hat{r}_1 + w_2\hat{r}_2 + \cdots + w_n\hat{r}_n = \sum_{i=1}^{n} w_i\hat{r}_i \tag{28-11}$$

Here the $\hat{r}_i$ values are the expected returns on the individual assets.

The variance and standard deviation of a portfolio depend not only on the variances and weights of the individual assets in the portfolio, but also on the correlation between the individual assets. The **correlation coefficient** between two assets i and j, ρ_{ij}, can range from −1.0 to +1.0. If the correlation coefficient is greater than 0, the assets are said to be **positively correlated,** while if the correlation coefficient is negative, they are **negatively correlated.**[10] Returns on positively correlated assets tend to move up and down together, while returns on negatively correlated assets tend to move in opposite directions. For a two-asset portfolio with assets 1 and 2, the portfolio standard deviation, σ_p, is calculated as follows:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}}. \tag{28-12}$$

Here $w_2 = (1 - w_1)$, and $\rho_{1,2}$ is the correlation coefficient between assets 1 and 2. Notice that if w_1 and w_2 are both positive, as they must be unless one asset is sold short, then the lower the value of $\rho_{1,2}$, the lower the value of σ_p. This is an important concept: Combining assets that have low correlations results in a portfolio with a low risk. For example, suppose the correlation between two assets is negative, so when the return on one asset falls, then that on the other asset will generally rise. The positive and negative returns will tend to cancel each other out, leaving the portfolio with very little risk. Even if the assets are not negatively correlated, but have a correlation coefficient less than 1.0, say 0.5, combining them will still be beneficial, because when the return on one asset falls dramatically, that on the other asset will probably not fall as much, and it might even rise. Thus, the returns will tend to balance each other out, lowering the total risk of the portfolio.

[10] See Chapter 3 for details on the calculation of correlations between individual assets.

To illustrate, suppose that in August 2003, an analyst estimates the following identical expected returns and standard deviations for Microsoft and General Electric:

	Expected Return, $\hat{r}$	Standard Deviation, σ
Microsoft	13%	30%
General Electric	13	30

Suppose further that the correlation coefficient between Microsoft and GE is $\rho_{M,GE} = 0.4$. Now if you have $100,000 invested in Microsoft, you will have a one-asset portfolio with an expected return of 13 percent and a standard deviation of 30 percent. Next, suppose you sell half of your Microsoft and buy GE, forming a two-asset portfolio with $50,000 in Microsoft and $50,000 in GE. (Ignore brokerage costs and taxes.) The expected return on this new portfolio will be the same 13.0 percent:

$$\begin{aligned}\hat{r}_p &= w_M\hat{r}_M + w_{GE}\hat{r}_{GE}\\ &= 0.50(13\%) + 0.50(13\%) = 13.0\%.\end{aligned}$$

Since the new portfolio's expected return is the same as before, what's the point of the change? The answer, of course, is that diversification reduces risk. As noted above, the correlation between the two companies is 0.4, so the portfolio's standard deviation is found to be 25.1 percent:

$$\begin{aligned}\sigma_p &= \sqrt{w_M^2\sigma_M^2 + w_{GE}^2\,\sigma_{GE}^2 + 2w_Mw_{GE}\sigma_M\sigma_{GE}\rho_{M,GE}}\\ &= \sqrt{(0.5)^2\,(0.3)^2 + (0.5)^2\,(0.3)^2 + 2(0.5)(0.5)(0.3)(0.3)(0.4)}\\ &= \sqrt{0.0630}\\ &= 0.251 = 25.1\%.\end{aligned}$$

Thus, by forming the two-asset portfolio you will have the same expected return, 13 percent, but with a standard deviation of only 25.1 percent versus 30 percent with the one-asset portfolio. So, because the stocks were not perfectly positively correlated, diversification has reduced your risk.

The numbers would change if the two stocks had different expected returns and standard deviations, or if we invested different amounts in each of them, or if the correlation coefficient were different from 0.4. Still, the bottom line conclusion is that, provided the stocks are not perfectly positively correlated, with $\rho_{1,2} = +1.0$, then diversification will be beneficial.[11]

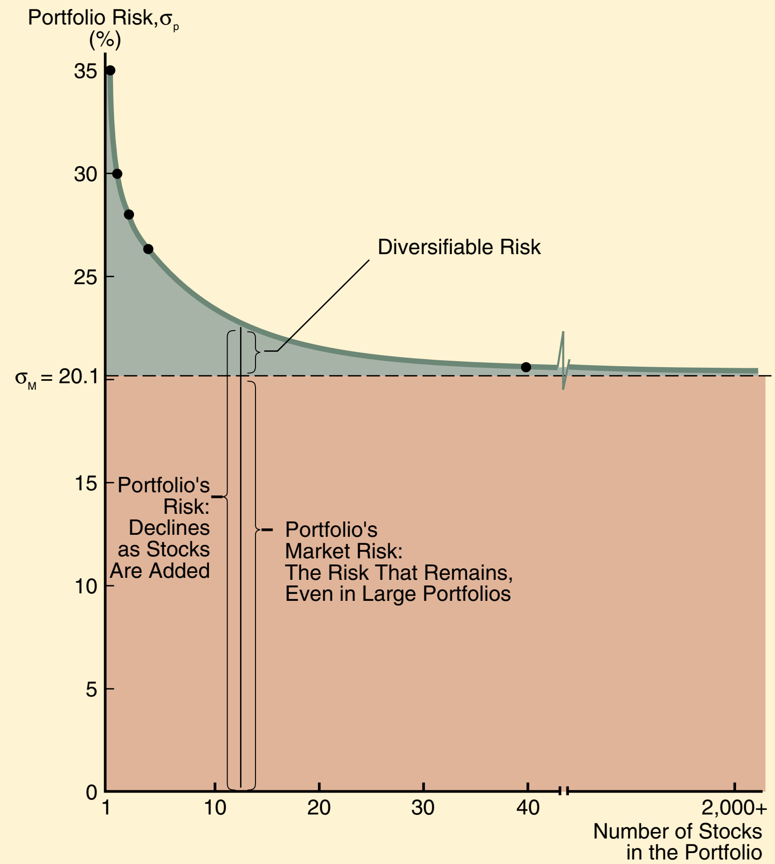
If a two-stock portfolio is better than a one-stock portfolio, would it be better to continue diversifying, forming a portfolio with more and more stocks? The answer is yes, at least up to some fairly large number of stocks. Figure 28-1 shows the relation between the number of stocks in a portfolio and the portfolio's risk for average NYSE stocks. Note that as the number of stocks in the portfolio increases, the total amount of risk decreases, but at a lower and lower rate, and it approaches a lower limit. This lower limit is called the **market risk** inherent in stocks, and no amount of diversification

[11] The standard deviation of a portfolio consisting of n assets with standard deviations σ_i, weights w_i and pairwise correlations $\rho_{i,j}$ is given by this equation:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_i\sigma_j\rho_{ij}}. \qquad \textbf{(28-13)}$$

Here $\rho_{i,j} = 1.0$. Note that this reduces to the two asset formula given above if n = 2.

Figure 28-1 Effects of Portfolio Size on Portfolio Risk for Average Stocks



can eliminate it. On the other hand, the risk of the portfolio in excess of the market risk is called **diversifiable risk,** and, as the graph shows, investors can reduce or even eliminate it by holding more and more stocks. It is not shown in the graph, but diversification among average stocks would not affect the portfolio's expected return—expected return would remain constant, but risk would decline as shown in the graph.

Investors who do not like risk are called **risk averse,** and such investors will choose to hold portfolios that consist of many stocks rather than only a few so as to eliminate the diversifiable risk in their portfolios. In developing the relationship between risk and return, we will assume that investors are risk averse, which implies that they will not hold portfolios that still have diversifiable risk. Instead, they will diversify their portfolios until only market risk remains. These resulting portfolios are called **well-diversified portfolios.**[12]

[12] If one selected relatively risky stocks, then the lower limit in Figure 28-1 would plot above the one shown for average stocks, and if the portfolios were formed from low-risk stocks, the lower limit would plot below the one we show. Similarly, if the expected returns on the added stocks differed from that of the portfolio, the expected return on the portfolio would be affected. Still, even if one wants to hold an especially high-risk, high-return portfolio, or a low risk and return portfolio, diversification will be beneficial.

Note too that holding more stocks involves more commissions and administrative costs. As a result, individual investors must balance these additional costs against the gains from diversification, and consequently most individuals limit their stocks to no more than 30 to 50. Also, note that if an individual does not have enough capital to diversify efficiently, then he or she can (and should) invest in a mutual fund.

The Capital Asset Pricing Model

Some investors have no tolerance whatever for risk, so they choose to invest all of their money in riskless Treasury bonds and receive a real return of about 3.5 percent.[13] Most investors, however, choose to bear at least some risk in exchange for an expected return that is higher than the risk-free rate. Suppose a particular investor is willing to accept a certain amount of risk in hope of realizing a higher rate of return. Assuming the investor is **rational,** he or she will choose the portfolio that provides the highest expected return for the given level of risk. This portfolio is by definition an **optimal** portfolio because it has the highest possible return for a given level of risk. But how can investors identify optimal portfolios?

One of the implications of the Capital Asset Pricing Model (CAPM) as discussed in Chapters 2 and 3 is that optimal behavior by investors calls for splitting their investments between the **market portfolio,** M, and a risk-free investment. The market portfolio consists of all risky assets, held in proportion to their market values. For our purposes, we consider an investment in long-term U.S. Treasury bonds to be a risk-free investment. The market portfolio has an expected return of $\hat{r}_M$ and a standard deviation of σ_M, and the risk-free investment has a guaranteed return of r_{RF}. The expected return on a portfolio with weight w_M invested in M, and weight w_{RF} (which equals $1.0 - w_M$) in the risk-free asset, is

$$\hat{r}_p = w_M\hat{r}_M + (1 - w_M)\, r_{RF}. \tag{28-11a}$$

Because a risk-free investment has zero standard deviation, the correlation term in Equation 28-12 is zero, hence the standard deviation of the portfolio reduces to

$$\sigma_p = w_M\sigma_M. \tag{28-12a}$$

These relationships show that by assigning different weights to M and to the risk-free asset, we will form portfolios with different expected returns and standard deviations. Combining Equations 28-11a and 28-12a, and eliminating w_M, we obtain this relationship between an optimal portfolio's return and its standard deviation:

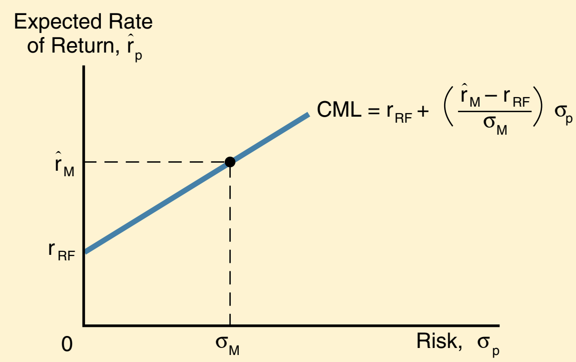
$$\hat{r}_p = r_{RF} + \left(\frac{\hat{r}_M - r_{RF}}{\sigma_M}\right)\sigma_p. \tag{28-14}$$

This equation is called the **Capital Market Line** (**CML**), and a graph of this relationship between risk and return is shown in Figure 28-2.

The CML shows the expected return that investors can expect at each risk level, assuming that they behave optimally by splitting their investments between the market portfolio and the risk-free asset. Note that the expected return on the market is greater than the risk-free rate, hence the CML is upward sloping. This means that investors who would like a portfolio with a higher rate of return must be willing to accept more risk as measured by

[13] Indexed T-bonds are essentially riskless, and they currently provide a real return of about 3.5 percent. This expected nominal return is 3.5 percent plus expected inflation.

Figure 28-2 The Capital Market Line (CML)



the standard deviation. Thus, investors who are willing to accept more risk are rewarded with higher expected returns as compensation for bearing this additional risk.

For example, suppose that $r_{RF} = 10\%$, $\hat{r}_M = 15\%$, and $\sigma_M = 20\%$. Under these conditions, a portfolio consisting of 50 percent in the risk-free asset and 50 percent in the market portfolio will have an expected return of 12.5 percent and a standard deviation of 10 percent. Varying the portfolio weights from 0 to 1.0 traces out the CML. Points on the CML to the right of the market portfolio ($\hat{r}_M$) can be obtained by putting portfolio weights on M greater than 1.0. This implies borrowing at the risk-free rate and then investing this extra money, along with the initial capital, in the market portfolio.

Beta

If investors are rational and thus hold only optimal portfolios (that is, portfolios that have only market risk and are on the CML), then the only type of risk associated with an individual stock that is relevant is *the risk the stock adds to the portfolio*. Refer to Figure 28-2 and note that investors should be interested in how much an additional stock moves the entire portfolio up or down the CML, not in how risky the individual stock would be if it were held in isolation. This is because some of the risk inherent in any individual stock can be eliminated by holding it in combination with all the other stocks in the portfolio. Chapters 2 and 3 show that the correct measure of an individual stock's contribution to the risk of a well-diversified portfolio is its **beta coefficient,** or simply **beta,** which is calculated as follows:

$$\text{Beta of stock i} = b_i = \frac{\rho_{i,M}\sigma_i\sigma_M}{\sigma_M^2} = \frac{\rho_{i,M}\sigma_i}{\sigma_M}. \quad (28\text{-}15)$$

Here $\rho_{i,M}$ is the correlation coefficient between Stock i and the market.

By definition, the market portfolio has a beta of 1.0. Adding a stock with a beta of 1.0 to the market portfolio will not change the portfolio's overall risk. Adding a stock with a beta of less than 1.0 will reduce the portfolio's risk, hence reduce its expected rate of return as shown in Figure 28-2. Adding a stock with a beta greater than 1.0 will increase the portfolio's risk and expected return. Intuitively, you can think of a stock's beta as a measure of how closely it moves with the market. A stock with a beta greater than 1.0 will tend to move up and down with the market, but with wider swings. A stock with a beta close to zero will tend to move independently of the market.

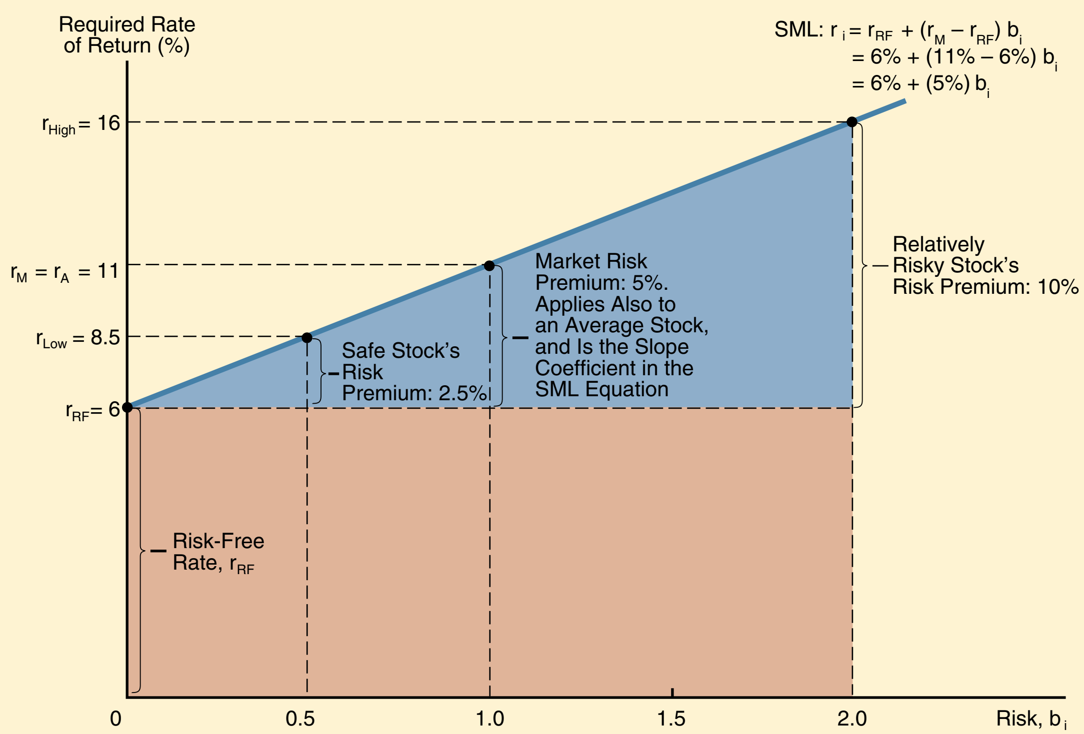
The CAPM shows the relationship between the risk that a stock contributes to a portfolio and the return that it must provide. The **required rate of return** on a stock is related to its beta by this formula:

$$\text{Required rate of return on Stock i} = r_i = r_{RF} + b_i(r_M - r_{RF}). \quad (28\text{-}16)$$

For given values of r_{RF} and r_M, the graph of r_i versus b_i is called the **Security Market Line (SML).** The SML shows the relationship between the required rate of return on a stock, its riskiness as measured by beta, and the required rate of return on the market. Figure 28-3 shows the SML and required rates of return for a low beta and a high beta stock. Note that the required rate of return on a stock is in excess of the risk-free rate, and it increases with beta. The extra return associated with higher betas is called the **risk premium,** and the risk premium on a given stock is equal to $b_i(r_M - r_{RF})$. The term $(r_M - r_{RF})$, which is called the **market risk premium,** or RP_M, amounts to the extra return an investor requires for bearing the market's risk.

The SML graph differs significantly from the CML graph. As Figure 28-2 shows, the CML defines the relationship between total risk, as measured by the standard deviation, and the expected rate of return for portfolios that are

Figure 28-3 **The Security Market Line (SML)**



combinations of the market portfolio and the risk-free asset. It shows the best available set of portfolios, based on risk and return, available to investors. The SML, on the other hand, shows the relationship between the required rate of return on individual stocks and their market risk as measured by beta.[14]

The Characteristic Line: Calculating Betas

Before we can use the SML to estimate a stock's required rate of return, we need to estimate the stock's beta coefficient. Recall that beta is a measure of how the stock tends to move with the market. Therefore, we can use the historical relationship between the stock's return and the market's return to calculate beta. First, note the following definitions:

$\bar{r}_J$ = historical (realized) rate of return on Stock J. (Recall that $\hat{r}_J$ and r_J are defined as Stock J's expected and required returns, respectively.)
$\bar{r}_M$ = historical (realized) rate of return on the market.
a_J = vertical axis intercept term for Stock J.
b_J = slope, or beta coefficient, for Stock J.
e_J = random error, reflecting the difference between the actual return on Stock J in a given year and the return as predicted by the regression line. This error arises because of unique conditions that affect Stock J but not most other stocks during a particular year.

The points on Figure 28-4 show the historical returns for Stock J plotted against historical market returns. The returns themselves are shown in the bottom half of the figure. The slope of the **regression line** that best fits these points measures the overall sensitivity of Stock J's return to the market return, and it is the beta estimate for Stock J. The equation for the regression line can be obtained by ordinary least squares analysis, using either a calculator with statistical functions or a computer with a regression software package such as a spreadsheet's regression function. In his 1964 article which first described the CAPM, Sharpe called this regression line the stock's **characteristic line.** Thus, a stock's beta is the slope of its characteristic line.

Figure 28-4 shows that the regression equation for Stock J is as follows:

$$\bar{r}_J = -8.9 + 1.6\,\bar{r}_M + e_J. \tag{28-17}$$

This equation gives the predicted future return on Stock J, given the market's performance in a future year.[15] So, if the market's return happens to be 20 percent next year, the regression equation predicts that Stock J's return will be $-8.9 + 1.6(20) = 23.1\%$.

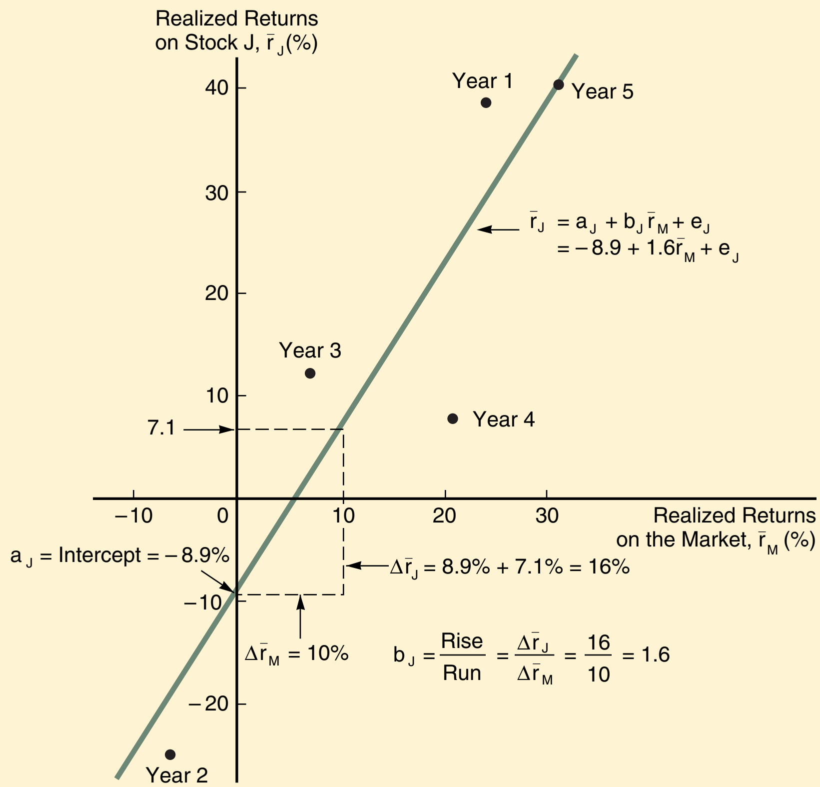
We can also use Equation 28-16 to determine the required rate of return on Stock J, given the required rate of return on the market and the risk-free rate as shown on the Security Market Line:

$$r_J = r_{RF} + 1.6(r_M - r_{RF}).$$

[14] Required rates of return are equal to expected rates of return as seen by the marginal investor if markets are efficient and in equilibrium. However, investors may disagree about the investment potential and risk of assets, in which case the required rate of return may differ from the expected rate of return as seen by an individual investor.

[15] This assumes no change in either the risk-free rate or the market risk premium.

Figure 28-4 Calculating Beta Coefficients



YEAR	STOCK J ($\bar{r}_J$)	MARKET ($\bar{r}_M$)
1	38.6%	23.8%
2	(24.7)	(7.2)
3	12.3	6.6
4	8.2	20.5
5	40.1	30.6
Average $\bar{r}$ =	14.9%	14.9%
$\sigma_{\bar{r}}$ =	26.5%	15.1%

Thus, if the risk-free rate is 8 percent and the required rate of return on the market is 13 percent, then the required rate of return on Stock J is 8% + 1.6(13% − 8%) = 16%.

Market versus Diversifiable Risk

Equation 28-17 can also be used to show how total, diversifiable, and market risk are related. The total risk for Stock J, σ_J^2, can be broken down into market risk and diversifiable risk:

$$\text{Total risk} = \text{Variance} = \text{Market risk} + \text{Diversifiable risk}.$$
$$\sigma_J^2 = b_J^2\sigma_M^2 + \sigma_{eJ}^2. \quad (28\text{-}18)$$

Here σ_J^2 is Stock J's variance (or total risk), b_J is the stock's beta coefficient, σ_M^2 is the variance of the market, and σ_{eJ}^2 is the variance of Stock J's regression error term. If all of the points in Figure 28-4 plotted exactly on the regression line, then J would have zero diversifiable risk and the variance of the error term, σ_{eJ}^2, would be zero. However, in our example all of the points do not plot exactly on the regression line, and the decomposition of total risk for J is thus

$$\begin{aligned}\text{Total risk} = 0.265^2 &= \text{Market risk} + \text{Diversifiable risk}\\ 0.0702 &= (1.6)^2(0.151)^2 + \sigma_{eJ}^2\\ 0.0702 &= 0.0584 + \sigma_{eJ}^2.\end{aligned}$$

Solving for σ_{eJ}^2 gives diversifiable risk of $\sigma_{eJ}^2 = 0.0118$. In standard deviation terms, the market risk is $\sqrt{0.0584} = 24.2\%$, and the diversifiable risk is $\sqrt{0.0118} = 10.9\%$.

Portfolio Betas

The beta of a portfolio can be calculated as the weighted average of the betas of the individual assets in the portfolio. This is in sharp contrast to the difficult calculation required in Equation 28-13 for finding the standard deviation of a portfolio. To illustrate the calculation, suppose an analyst has determined the following information for a four-stock portfolio:

Stock (1)	Weight (2)	Beta (3)	Product (4) = (2) × (3)
I	0.40	0.6	0.24
J	0.20	1.0	0.20
K	0.30	1.3	0.39
L	0.10	2.1	0.21
	1.00		Portfolio beta: 1.04

The beta is calculated as the sum of the product terms in the table. We could also calculate it using this equation:

$$b_p = 0.4(0.6) + 0.2(1.0) + 0.3(1.3) + 0.1(2.1) = 1.04.$$

Here the first value in each term is the stock's weight in the portfolio, the second term is the stock's beta, and the weighted average is the portfolio's beta, $b_p = 1.04$.

Now assume that the risk-free rate is 8 percent, and the required rate of return on the market portfolio is 12 percent. The portfolio's required return can be found using Equation 28-16:

$$r_p = 8\% + 1.04(12\% - 8\%) = 12.16\%.$$

SELF-TEST QUESTIONS

How are the expected return and standard deviation calculated from a probability distribution?
When is the coefficient of variation a useful measure of risk?
What is the difference between diversifiable risk and market risk?
Explain the following statement: "An asset held as part of a portfolio is generally less risky than the same asset held in isolation."
What does it mean to say that beta is the theoretically correct measure of a stock's riskiness?
What is the difference between the CML and the SML?
How would you calculate a beta?
What is optimal about an "optimal portfolio"?

Stock Valuation

The techniques used earlier in this chapter to value bonds can also be used, with a few modifications, to value stocks. First, note that the cash flows from stocks that companies provide to investors are dividends rather than coupon payments, and there is no maturity date on stocks. Moreover, dividend payments are not contractual, and they typically are expected to grow over time, not to remain constant as bond interest payments do. Here is the general equation used to value common stocks:

$$\text{Price} = \text{PV} = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \cdots + \frac{D_n}{(1+r)^n} + \cdots = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}. \quad \text{(28-19)}$$

Here the D_t terms represent the dividend expected in each year t, and r is the stock's required rate of return as determined in the preceding section. There is no maturity date, so the present value must be for all expected dividend payments extending out forever.

Equation 28-19 for stocks differs from Equation 28-6 for bonds in that D_t represents an *expected but uncertain, and nonconstant,* dividend rather than a fixed, known coupon or principal payment, and also because the summation goes out to infinity. The required rate of return on the stock, r, could either be determined by the CAPM or just estimated subjectively by the investor.

Dividend Growth Model

Rather than explicitly projecting dividends forever, under certain assumptions we can simplify the process. In particular, if our best guess is that the firm's earnings and dividends will grow at a constant rate g on into the foreseeable future, then we can use the **dividend growth model.** For example, suppose a company has just paid a dividend of $1.50, and its dividends are projected to grow at a rate of 6 percent per year. The expected dividend at the end of the current year will be $1.50(1.06) = $1.59. The dividend expected during the second year will be $\$1.50(1.06)^2 = \$1.6854$, and the dividend expected during the t'th year will be $\$1.50(1.06)^t$.

We can replace D_t with $D_0(1 + g)^t$ in Equation 28-19, in which case we have a power series that can be solved to give the following formula:[16]

$$\text{Price now} = \hat{P}_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}. \quad \text{(28-20)}$$

This equation is called the **constant growth model,** or the **Gordon model** after Myron J. Gordon, who did much to develop and popularize it. In our example, $D_0 = \$1.50$ and $g = 0.06$. If the required rate of return on the stock is 13 percent, then the present value of all expected future dividends will be 1.50(1.06)/(0.13 − 0.06) = $22.71, and this is the value of the stock according to the model.

[16] See Chapter 5 for the derivation of Equation 28-20.

Equation 28-20 also provides an alternative to the CAPM for estimating the required rate of return on a stock. First, we transform Equation 28-20 to form Equation 28-21:

$$\hat{r}_s = D_1/P_0 + g. \tag{28-21}$$

Here we write the rate of return variable as $\hat{r}_s$ rather than r to indicate that it is an expected as opposed to a required rate of return. The equation indicates that the expected rate of return on a stock whose dividend is growing at a constant rate, g, is the sum of its expected dividend yield and its constant growth rate. In equilibrium, this is also the required rate of return.

Both the current price and the most recent dividend can be readily determined, and analysts make and publish growth rate estimates. Thus, given the input data, we can solve for $\hat{r}_s$. Here is the situation for our illustrative stock:

$$\hat{r}_s = D_1/P_0 + g = \$1.50(1.06)/\$22.71 + 0.06 = 0.13 = 13\%.$$

In Chapter 10 we discuss the types of firms for which this analysis is appropriate. We also provide other simplified models in the next two sections for use in situations where the assumption of constant growth is not appropriate.

Perpetuities

A second simplified version of the general stock valuation model as expressed in Equation 28-19 can be used to value **perpetuities,** which are securities that are expected to pay a constant amount each period forever. Preferred stock is an example of a perpetuity. Many preferred stocks have no maturity date, and they pay a constant dividend. In this case $g = 0$ in Equation 28-20, and the expression reduces to $P = D/r$. A share of \$3 preferred stock (i.e., the stock pays \$3 every year) with a required rate of return of 9 percent will sell for \$3.00/0.09 = \$33.33. The perpetuity formula also makes it easy to find the expected yield on a security, given the dividend and the current price: $\hat{r}_s = D/P$. So, if a share of preferred stock that pays an annual dividend of \$4 per share trades for \$65 per share, the stock's yield is \$4/\$65 = 6.15%.

Nonconstant Growth Model

Often it is inappropriate to assume that a company's dividends will grow at a constant rate forever. Most firms start out small and pay no dividends during their initial growth phase. Later, when they are able, they start paying a small dividend, and they then increase this dividend relatively rapidly, as earnings continue to grow. Finally, as the firm begins to mature, its dividend growth rate declines to the overall growth rate of the company or the industry. Thus, the growth rate is initially zero, then it becomes positive and large, and finally it declines and approaches a constant rate. In this situation, the dividend growth model given in Equation 28-20 is not appropriate, and an attempt to use it would give inaccurate and misleading results.

The best way to deal with a changing dividend growth rate is to use the **nonconstant growth model.** This involves several steps:

- Estimate each dividend during the nonconstant growth period.
- Determine the present value of each of the nonconstant growth dividends.

- Use the constant growth model to find the discounted value of all of the dividends expected once the nonconstant growth period has ended, which is the expected stock price at the end of the nonconstant growth period.
- Find the present value of the expected future price.
- Sum the PVs of the nonconstant dividends and the PV of the expected future price to find the value of the stock today.

For example, suppose the required rate of return on a stock is 12 percent. A dividend of $0.25 has just been paid, and you expect the dividend to double each year for four years. After Year 4, the firm will reach a steady state and have a dividend growth rate of 8 percent per year forever. The following time line shows the cash flows:

	0		1		2		3		4		5		. . . ∞
Growth rate		100%		100%		100%		100%		8%		8%	
Dividend			0.50		1.00		2.00		4.00		4.32		

To begin, let's find the price at the end of the fourth year, at which point we will have a constant growth stock. Someone buying the stock will receive D_5 and the subsequent dividends, which will presumably grow at a constant rate of 8 percent forever. Equation 28-20 can be used to find $\hat{P}_4$:

$$\text{Price at end of Year 4} = \hat{P}_4 = \frac{D_5}{r - g}. \tag{28-20a}$$

This means that the constant growth rate model can be used *as of Year 4,* and the stock price expected in that year is found as follows:

$$\hat{P}_4 = \frac{D_5}{r - g} = \frac{4.32}{0.12 - 0.08} = \$108. \tag{28-20b}$$

This gives us the following time line that stops at the end of Year 4:

	0	1	2	3	4
Dividend		0.50	1.00	2.00	4.00
Stock price					108.00
Total cash flow		0.50	1.00	2.00	112.00

The present value of these four cash flows at the 12 percent required rate of return is

$$\begin{aligned}\hat{P}_0 &= \text{Present value of cash flows} \\ &= \frac{0.50}{1.12} + \frac{1.00}{1.12^2} + \frac{2.00}{1.12^3} + \frac{112.00}{1.12^4} = \$73.85.\end{aligned}$$

Thus, the stock should trade for $73.85 today, and it is expected to rise to $108 four years from now.

For some firms, especially startups and other small companies, it is more reasonable to expect the company to be acquired by a larger company than to continue forever as an independent operation. In such cases, we can modify the nonconstant growth model by using a terminal value based on the expected acquisition price rather than a price based on the constant growth model. We discuss procedures for valuing such companies in Chapters 10 and 25.

SELF-TEST QUESTIONS

Is a higher percentage of a stock's value based on this year's earnings and dividends or on a forecast of long-term earnings and dividends?
Write out and explain the valuation model for a constant growth stock and for a perpetuity.
How do you value a stock that is not expected to grow at a constant rate?
How can you use the constant growth model to find the required rate of return on a stock?

SUMMARY

The goal of this chapter was to review the fundamental tools of (1) time value of money, (2) risk and return, and (3) valuation models for stocks and bonds. The key concepts covered are listed below.

- The **future value of a single payment** is $FV_n = PV(1 + i)^n = PV(FVIF_{i,n})$.
- The **future value of an annuity** is

$$\begin{aligned}\text{Future value} = FVA_n &= PMT(1 + i)^{n-1} + PMT(1 + i)^{n-2} + \cdots + PMT(1 + i) + PMT \\ &= PMT\left[\frac{(1 + i)^n - 1}{i}\right] \\ &= PMT(FVIFA_{i,n}).\end{aligned}$$

- The **present value of a single payment** is $PV = FV_n/(1 + i)^n$.
- The **present value of an annuity** is

$$\begin{aligned}\text{Present value} = PVA_n &= \frac{PMT}{(1 + i)} + \frac{PMT}{(1 + i)^2} + \frac{PMT}{(1 + i)^3} + \ldots + \frac{PMT}{(1 + i)^{n-1}} + \frac{PMT}{(1 + i)^n} \\ &= PMT\left[\frac{1 - \frac{1}{(1 + i)^n}}{i}\right] \\ &= PMT(PVIFA_{i,n}).\end{aligned}$$

- The **effective annual rate** for m compounding intervals per year at a nominal rate of i_{nom} per year is EAR (or EFF%) $= \left(1 + \frac{i_{nom}}{m}\right)^m - 1.0$.
- The effective annual rate for **continuous compounding** at a nominal rate of i_{nom} per year is $EAR_{continuous} = e^{i_{nom}} - 1.0$.
- If you know the cash flows and the PV (or FV) of a cash flow stream, you can **determine the interest rate** using a financial calculator to solve for the interest rate.
- The general valuation equation for a series of cash flows is

$$PV = \frac{CF_1}{1 + i} + \frac{CF_2}{(1 + i)^2} + \cdots + \frac{CF_n}{(1 + i)^n} = \sum_{t=1}^{n} \frac{CF_t}{(1 + i)^t}.$$

- The price of a bond is the present value of its coupon and principal payments:

$$\text{Price of bond} = \sum_{t=1}^{N} \frac{\text{Coupon}}{(1 + r_d)^t} + \frac{\text{Par}}{(1 + r_d)^N}.$$

- The riskiness of an asset's cash flows can be considered either on a **stand-alone basis,** with each asset thought of as being held in isolation, or in a **portfolio context,** where the investment is combined with other assets and its risk is reduced through **diversification.**
- The **relevant risk** of an individual asset is its contribution to the riskiness of a well-diversified **portfolio,** which is the asset's **market risk.** This risk is measured by **beta.** Since market risk cannot be eliminated by diversification, investors must be compensated for bearing it.
- The **Security Market Line (SML)** equation shows the relationship between a security's risk and its required rate of return. The return required on any security is equal to the risk-free rate plus the market risk premium times the security's beta:

$$r_i = r_{RF} + b_i(r_M - r_{RF}).$$

- The **Capital Market Line** describes the risk/return relationship for optimal portfolios, that is, for portfolios that consist of a mix of the market portfolio and a riskless asset.
- The beta coefficient is measured by the slope of the stock's **characteristic line,** which is found by regressing historical returns on the stock versus historical returns on the market.
- The **value of a share of stock** is calculated as the **present value of the stream of dividends** the stock is expected to provide in the future.
- The equation used to find the **value of a constant growth stock** is

$$\hat{P}_0 = \frac{D_0(1 + g)}{r - g} = \frac{D_1}{r - g}.$$

- The equation for $\hat{r}_s$, the **expected rate of return on a constant growth stock,** can be expressed as follows: $\hat{r}_s = D_1/P_0 + g$.
- To find the **present value of a supernormal growth stock** (1) find the dividends expected during the supernormal growth period, (2) find the price of the stock at the end of the supernormal growth period, (3) discount the dividends and the projected price back to the present, and (4) sum these PVs to find the stock's value, $\hat{P}_0$.
- The value of a **perpetuity** can be found using the constant growth formula with $g = 0$.

QUESTIONS

(28-1) Define each of the following terms:

a. PV; i or I; FV_n; PMT; m; i_{nom}
b. $FVIF_{i,n}$; $PVIF_{i,n}$; $FVIFA_{i,n}$; $PVIFA_{i,n}$
c. Equivalent Annual Rate (EAR); nominal (quoted) interest rate
d. Amortization schedule; principal component versus interest component of a payment
e. Par value; maturity date; coupon payment; coupon interest rate
f. Premium bond; discount bond
g. Stand-alone risk; risk; probability distribution
h. Expected rate of return, $\hat{r}$
i. Risk Premium for Stock i, RP_i; market risk premium, RP_M
j. Capital Asset Pricing Model, CAPM
k. Market risk; diversifiable risk; relevant risk
l. Beta coefficient, b
m. Security Market Line, SML
n. Optimal (or efficient) portfolio

o. Capital Market Line, CML
p. Characteristic line
q. Intrinsic value, $\hat{P}_0$; market price, P_0
r. Required rate of return, r_S; expected rate of return, $\hat{r}_S$; actual, or realized, rate of return, $\bar{r}_S$.
s. Normal, or constant, growth; supernormal, or nonconstant, growth

(28-2) Would you rather have a savings account that pays 5 percent interest compounded semiannually, or one that pays 5 percent interest compounded daily? Explain.

(28-3) The rate of return you would earn if you bought a bond and held it to its maturity date is called the bond's *yield to maturity*. How is the yield to maturity related to overall interest rates in the economy? If interest rates in the economy fall after a bond has been issued, what will happen to the bond's yield to maturity and price? Will the size of any changes be affected by the bond's maturity?

(28-4) Security X has an expected return of 6 percent, a standard deviation of expected returns of 40 percent, a correlation coefficient with the market of −0.20, and a beta coefficient of −0.4. Security Y has an expected return of 13 percent, a standard deviation of returns of 25 percent, a correlation with the market of 0.8, and a beta coefficient of 1.0. Which security is riskier? Why?

(28-5) If a stock's beta were to drop to half of its former level, would its expected return also drop by half?

(28-6) What is the difference between the SML and the CML?

(28-7) Two investors are evaluating GE's stock for possible purchase. They agree on the expected value of D_1 and also on the expected future dividend growth rate. Further, they agree on the riskiness of the stock. However, one investor normally holds stocks for 2 years, while the other normally holds stocks for 10 years. On the basis of the type of analysis done in this chapter, should they be willing to pay the same price for GE's stock? Explain.

PROBLEMS

(28-1) Future Value of an Annuity

Find the *future value* of the following annuities. The first payment is made at the *end* of Year 1, so they are *ordinary annuities:*

a. \$500 per year for 8 years at 9%.
b. \$300 per year for 6 years at 4%.
c. \$500 per year for 6 years at 0%.
d. Now rework parts a, b, and c assuming that payments are made at the *beginning* of each year; that is, they are *annuities due*.

(28-2) Present Value of an Annuity

Find the *present value* of the following *ordinary annuities:*

a. \$500 per year for 8 years at 9%.
b. \$300 per year for 6 years at 4%.
c. \$500 per year for 6 years at 0%.
d. Now rework parts a, b, and c assuming that payments are made at the *beginning* of each year; that is, they are *annuities due*.

(28-3) Effective Rate of Interest

Find the interest rates, or rates of return, on each of the following:

a. You *borrow* \$900 and promise to pay back \$972 at the end of 1 year.
b. You *lend* \$900 and receive a promise to be paid \$972 at the end of 1 year.
c. You borrow \$65,000 and promise to pay back \$310,998 at the end of 15 years.
d. You borrow \$11,000 and promise to make payments of \$2,487.22 per year for 7 years.

(28-4) PV and Effective Annual Yield

Your grandmother has asked you to evaluate two alternative investments for her. The first is a security that pays $50 at the end of each of the next 3 years, with a final payment of $1,050 at the end of Year 4. This security costs $900. The second investment involves simply putting the same amount of money in a bank savings account that pays an 8 percent nominal (quoted) interest rate, but with quarterly compounding. Your grandmother regards the two investments as being equally safe and liquid, so the required effective annual rate of return on the security is the same as that on the bank deposit. She has asked you to calculate the value of the security to help her decide whether it is a good investment. What is its value relative to the bank deposit?

(28-5) Required Annuity Payments

Assume that your father is now 55 years old, that he plans to retire in 12 years, and that he expects to live for 20 years after he retires, that is, until he is 87. He wants a fixed retirement income that has the same purchasing power at the time he retires as $60,000 has today (he realizes that the real value of his retirement income will decline year by year after he retires, but he wants level payments during retirement anyway). His retirement income will begin the day he retires, 12 years from today, and he will receive 20 annual payments. Inflation is expected to be 5 percent per year from today forward. He currently has $100,000 in savings, and he expects to earn a return on his savings of 8 percent per year, annual compounding. To the nearest dollar, how much must he save during each of the next 12 years (with deposits being made at the end of each year) to meet his retirement goal?

(28-6) Yield to Maturity

Cargill Diggs' bonds have 15 years remaining to maturity. Interest is paid annually, the bonds have a $1,000 par value, and the coupon interest rate is 9.5 percent. The bonds sell at a price of $850. What is their yield to maturity?

(28-7) Bond Valuation

The Peabody Company has two bond issues outstanding. Both pay $110 annual interest plus $1,000 at maturity. Bond H has a maturity of 14 years and Bond K a maturity of 2 years.

a. What is the value of each of these bonds if the going rate of interest is (1) 5 percent, (2) 8 percent, and (3) 12 percent? Assume that two more interest payments will be made on Bond K and 14 more on Bond H.

b. Why does the longer-term (14-year) bond fluctuate more when interest rates change than does the shorter-term bond (2-year)?

(28-8) Bond Valuation

Suppose Integon Inc. sold an issue of bonds with a 10-year maturity, a $1,000 par value, a 9 percent coupon rate, and semiannual interest payments.

a. Two years after the bonds were issued, the going rate of interest on bonds such as these fell to 7 percent. At what price would the bonds sell?

b. Suppose that, 2 years after the initial offering, the going interest rate had risen to 11 percent. At what price would the bonds sell?

c. Suppose that the conditions in part a existed—that is, interest rates fell to 7 percent 2 years after the issue date. Suppose further that the interest rate remained at 7 percent for the next 8 years. What would happen to the price of the Integon bonds over time? (Hint: How much should one of these bonds sell for just before maturity?)

(28-9) Interest Rate Risk

A bond trader purchased each of the following bonds at a yield to maturity of 9 percent. Immediately after she purchased the bonds, interest rates fell to 8 percent. To show the price sensitivity of each bond to changes in interest rates, fill in the blanks in the following table:

Price @ 9%	Price @ 8%	Percentage Change
10-year 10% annual coupon	_____	_____
10-year zero coupon	_____	_____
5-year zero coupon	_____	_____
30-year zero coupon	_____	_____
$100 perpetuity	_____	_____

Could you use the percentage change as a measure of the bonds' interest rate risk? Would you be making any assumptions about the shape of the yield curve if you did this? Might changes in the shape of the yield curve affect changes in the bonds' prices in the real world? Discuss how the yield curve might affect a bond's price variability over time.

(28-10) Bond Valuation

An investor has two bonds in his portfolio. Each bond matures in 3 years, has a face value of $1,000, and has a yield to maturity of 11.1 percent. Bond A pays an annual coupon of 13 percent. Bond Z is a zero coupon bond. Assuming the yield to maturity of each bond remains at 11.1 percent over the next 4 years, what will be the price of each of the bonds at the following time periods? Fill in the following table:

t	Price of Bond A	Price of Bond Z
0	_____	_____
1	_____	_____
2	_____	_____
3	_____	_____

(28-11) Expected Return

A stock's expected return has the following distribution:

Demand for the Company's Products	Probability of This Demand Occurring	Rate of Return if This Demand Occurs
Weak	0.2	(40%)
Below average	0.2	(10)
Average	0.1	8
Above average	0.3	20
Strong	0.2	50

Calculate the stock's expected return, standard deviation, and coefficient of variation.

(28-12) Portfolio Beta

An individual has $60,000 invested in a stock that has a beta of 1.3 and $15,000 invested in a stock with a beta of 0.6. If these are the only two investments in her portfolio, what is her portfolio's beta?

(28-13) Required Rate of Return

Suppose $r_{RF} = 9\%$, $r_M = 14\%$, and $r_A = 20\%$.

a. Calculate Stock A's beta.

b. If Stock A's beta changed to 1.5, how would the required rate of return change?

(28-14) Required Rate of Return

Suppose $r_{RF} = 6\%$, $r_M = 11\%$, and $b_i = 1.3$.

a. What is r_i, the required rate of return on Stock i?

b. Now suppose r_{RF} (1) increases to 7 percent or (2) decreases to 5 percent. The slope of the SML remains constant. How would this affect r_M and r_i?

c. Now assume that r_{RF} remains at 6 percent but r_M (1) increases to 13 percent or (2) falls to 10 percent. The slope of the SML does *not* remain constant. How would these changes affect r_i?

(28-15) Portfolio Beta

You have a $3 million portfolio consisting of a $150,000 investment in each of 20 different stocks. The portfolio has a beta of 1.2. You are considering selling $150,000 worth of one stock that has a beta equal to 0.8 and using the proceeds to purchase a stock that has a beta equal to 1.4. What will be the new beta of your portfolio following this transaction?

(28-16) Characteristic Line and SML

You are given the following set of data:

	HISTORICAL RATES OF RETURN	
Year	NYSE	Stock ABC
1	(26.5%)	(14.5%)
2	37.2	23.5
3	23.8	17.0
4	(7.2)	2.5
5	6.6	7.6
6	20.5	19.9
7	30.6	18.2

a. Use a calculator with a linear regression function (or a spreadsheet) to determine Stock ABC's beta coefficient, or else plot these data points on a scatter diagram, draw in the regression line, and then estimate the value of the beta coefficient "by eye."
b. Determine the arithmetic average rates of return for Stock ABC and the NYSE over the period given. Calculate the standard deviations of returns for both Stock ABC and the NYSE.
c. Assuming (1) that the situation during Years 1 to 7 is expected to hold true in the future (that is, $\hat{r}_{ABC} = \bar{r}_{ABC}$; $\bar{r}_M = \hat{r}_M$; and both σ_{ABC} and b_{ABC} in the future will equal their past values), and (2) that Stock ABC is in equilibrium (that is, it plots on the SML), what is the implied risk-free rate?
d. Plot the Security Market Line.
e. Suppose you hold a large, well-diversified portfolio and are considering adding to the portfolio either Stock ABC or another stock, Stock Y, that has the same beta as Stock ABC but a higher standard deviation of returns. Stocks ABC and Y have the same expected returns; that is $\hat{r}_{ABC} = \hat{r}_Y = 10.6\%$. Which stock should you choose?

(28-17) SML and CML Comparison

The beta coefficient of an asset can be expressed as a function of the asset's correlation with the market, its standard deviation, and the market's standard deviation:

$$b_i = \frac{\rho_{i,M}\sigma_i}{\sigma_M}$$

a. Substitute this expression for beta into the Security Market Line (SML), Equation 28-16. This results in an alternative form of the SML.
b. Compare your answer to part a with the Capital Market Line (CML), Equation 28-14. What similarities are observed? What conclusions can be drawn?

(28-18) Constant Growth Valuation

Axxon Inc. is expected to pay a \$0.75 per share dividend at the end of the year (i.e., D_1 = \$0.75). The dividend is expected to grow at a constant rate of 7 percent a year. The required rate of return on the stock, r_s, is 15 percent. What is the value of a share of the company's stock?

(28-19) Constant Growth Valuation

Pic-A-Shoe's stock currently sells for \$25. The stock just paid a dividend of \$1.25, that is, D_0 = \$1.25. The dividend is expected to grow at a constant rate of 8 percent a year. What stock price is expected 1 year from now? What is the required rate of return?

(28-20) Declining Growth Stock Valuation

Escrow Oil's petroleum reserves are being depleted, so its sales revenues are falling. Also, its reserves are becoming increasingly deep each year, so the costs of bringing oil to the surface are rising. As a result, the company's earnings and dividends are declining at a constant rate of 5 percent per year. If D_0 = \$6 and r_S = 15%, what is the value of Escrow Oil's stock?

(28-21) Supernormal Growth Stock Valuation

Assume that an average firm in your company's industry is expected to grow at a constant rate of 7 percent and has a dividend yield of 6 percent. Your company is of average risk, but it has just successfully completed some R&D work which leads you to expect that its earnings and dividends will grow at a rate of 50 percent [$D_1 = D_0(1 + g) = D_0(1.50)$] this year, and 25 percent the following year, after which growth should match the 7 percent industry average rate. The last dividend paid (D_0) was $2.00. What is the value per share of your firm's stock today? (Hint: use Equation 28-21 to find the required rate of return.)

(28-22) Preferred Stock Valuation

Jonesboro Corporation issued preferred stock with a stated dividend of 9 percent of par. Preferred stock of this type currently yields 7 percent, and the par value is $100. Assume dividends are paid annually.

a. What is the value of Jonesboro Corporation's preferred stock?
b. Suppose interest rate levels rise to the point where preferred stock yields 12 percent. Now what would be the value of Jonesboro's preferred stock?

CYBERPROBLEM

Please go to our web site, **http://brigham.swlearning.com**, to access the Cyberproblems.

THOMSON ANALYTICS™

With your Xtra! CD-ROM, access the Thomson Analytics Problems and use the Thomson Analytics Academic online database to work this chapter's problems.

SPREADSHEET PROBLEM

(28-23) Build a Model: Loan Amortization

Start with the partial model in the file ***Ch 28 P23 Build a Model.xls*** from the textbook's Student CD or web site. Set up an amortization schedule for a $30,000 loan to be repaid in equal installments at the end of each of the next 20 years at an interest rate of 10 percent.

a. What is the annual payment?
b. Set up an amortization schedule for a $60,000 loan to be repaid in 20 equal annual installments at an interest rate of 10 percent. What is the annual payment?
c. Set up an amortization schedule for a $60,000 loan to be repaid in 20 equal annual installments at an interest rate of 20 percent. What is the annual payment?

MINI CASE

Susan Greene is a financial planner. Her job is to suggest and implement investment and savings plans for clients, some of whom are of modest means and some of whom are quite wealthy. Last month a *Fortune* 500 firm contracted with Susan's firm to provide financial planning services to all 150 of its middle level managers over a 3-month period. Susan's plan is first to conduct an hour-long seminar to provide some general information about investments and their risks, and then to arrange individual meetings for follow-up. She has asked you to help her with the seminar by working out some examples to illustrate savings and investment plans, and to prepare some information about the risks and rewards of investments in common stock. She particularly wants you to explain how risk and return are related. Please answer the following questions and prepare the following illustrations for her.

a. Draw time lines (1) for a $100 lump sum due at the end of Year 2 and (2) for a 3-year $100 annuity. Explain how each investment of $100 grows to its future value after 3 years if the interest rate is 10 percent.

b. What is the present value of the $100 lump sum due at the end of 2 years and the 3-year, $100 annuity in part a? How much would you need to invest in an account that earns 10 percent in order to fund this annuity?

c. If inflation is 3 percent per year, then an annual salary of $60,000 today will rise to about $145,000 in 30 years. One of Susan's clients wants to maintain a purchasing power of $60,000 in today's dollars, at least for the first year of retirement, so she is planning to draw $145,000 a year at year-end after retirement. How much must she save monthly to retire in 30 years and draw an annual pension of $145,000 for 20 years after retirement? Assume a 10 percent per year return on investments.

d. Susan has also asked you to prepare some information about various investments that might be used to meet these retirement goals. To do this you will need to discuss not only how bonds and stocks are priced but also the concept of the trade-off between risk and return. To begin, describe the key features of a bond and show how its value is determined. Find the value of a 10-year, $1,000 par value bond with a 10 percent annual coupon and a required rate of return of 10 percent.

e. (1) What would be the value of the bond described in part d if, just after it had been issued, the expected inflation rate rose by 3 percentage points, causing investors to require a 13 percent return?

(2) What would happen to the bond's value if inflation fell, causing r_d to decline to 7 percent?

(3) What would happen to the value of the 10-year bond over time if the required rate of return remained at 13 percent? If it remained at 7 percent?

f. As an alternative to bond investments, Susan wants you to present some data on stock investments. The following table has the returns that should occur under various states of the economy for a variety of assets. Some of the output variables have been calculated, but blanks are shown for others.

(1) Calculate the expected return, standard deviation, and CV for HT and the T-bills.

(2) How do HT's return, standard deviation, and CV compare with those of the other assets, and what are the implications of these comparisons?

g. Suppose you created a 2-stock portfolio by investing $50,000 in HT and $50,000 in Collections.

(1) Calculate the expected return (r_P), the standard deviation (σ_p) and coefficient of variation (CV_p) for this portfolio.

(2) How does the riskiness of this portfolio compare to the riskiness of the individual stocks if held in isolation?

h. Explain what happens to the risk and expected return on a portfolio constructed from randomly picked stocks if we start with a 1-stock portfolio and add more and more stocks.

i. How are risk and return related under the CAPM? Specifically, how is beta calculated and how are required rates of return determined? Use the SML to calculate required rates of return for the three stocks in part f. How do these required returns compare with the stocks' expected returns?

j. Explain why the price of a share of stock is calculated as the present value of its expected future dividends, using a time line to help with your explanation.

k. One of Susan's clients has just inherited some stock of a company named Bon Temps, and he asked her to evaluate the stock for him. Use the dividend growth model to find the price of a share of Bon Temps stock. Bon Temps has a beta coefficient of 1.2, the risk-free rate is 7 percent, the required rate of return on the market is 12 percent. Bon Temps is a constant growth firm whose last dividend (D_0) was $2 and whose dividend is expected to grow at a rate of 6 percent indefinitely.

l. If Bon Temps were selling for $30.29, what would be its implied expected rate of return?

m. Now assume that Bon Temps is expected to experience supernormal growth of 30 percent for the next 3 years, then to return to its long-run constant growth rate of 6 percent. What is the stock's value under these conditions?

ESTIMATED RATE OF RETURN

			STOCKS			
State of the Economy	Probability	T-Bills	HT	Collections	USR	Market Portfolio
Recession	0.1	8.0%	(22.0%)	28.0%	10.0%	(13.0%)
Below Average	0.2	8.0	(2.0)	14.7	(10.0)	1.0
Average	0.4	8.0	20.0	0.0	7.0	15.0
Above Average	0.2	8.0	35.0	(10.0)	45.0	29.0
Boom	0.1	8.0	50.0	(20.0)	30.0	43.0
$\hat{r}$				1.7%	13.8%	15.0%
σ		0		13.4	18.8	
CV				7.7	1.4	
b			1.30	−0.87	0.89	1.00

SELECTED ADDITIONAL REFERENCES AND CASES

For a more complete discussion of the mathematics of finance, see

Atkins, Allen B., and Edward A. Dyl, "The Lotto Jackpot: The Lump Sum versus the Annuity," *Financial Practice and Education,* Fall/Winter 1995, 107–111.

Cissell, Robert, Helen Cissell, and David C. Flaspohler, *Mathematics of Finance,* 8th ed. (Boston: Houghton Mifflin, 1990).

Lindley, James T., "Compounding Issues Revisited," *Financial Practice and Education,* Fall 1993, 127–129.

Many investment textbooks cover risk and return concepts and bond and stock valuation models in depth and detail. Some of the better ones are listed in the Chapter 2 references. Additional references for bond valuation appear in the Chapter 4 references. Additional references for stock valuation appear in the Chapter 5 references. Additional references on CAPM theory appear in the Chapter 3 references.

The following cases from the Finance Online Case Library *cover many of the concepts discussed in this chapter and are available at* ***http://www.textchoice.com:***

Case 2, "Peachtree Securities, Inc. (A)," which deals with risk and return and CAPM concepts.

Case 3, "Peachtree Securities, Inc. (B)," which focuses on valuation concepts.

Case 56, "Laura Henderson," which focuses on valuation, risk, and portfolio construction.